Effect of Launch Angle on Horizontal Distance Travelled by a Projectile

INTRODUCTION

Through derivation, 45° is undoubtedly the best launch angle in order for a projectile to go the furthest horizontal distance (*How should you launch a ball to achieve the greatest distance?*, 2010). It can include activities such as long jump, throwing a football, and kicking a soccer ball, which theoretically reach the furthest horizontal distance at 45°. In real world conditions there exists air resistance, and the angle to launch the projectile the furthest distance will be different. In this experiment, I will investigate the effect that different launch angles have on the distance travelled by a marble with a homemade launcher launched with a spring in the presence of air resistance.

EQUATION

The relation between the horizontal distance travelled (d_x) and launch angle (θ) should be a parabola with the equation given below, while the distance (d_x) is linear to sin2θ.

$$d_x = \frac{FX}{mg} sin2\theta$$

Where F is the force exerted by the spring (N), X is the distance the spring is compressed (m), θ is the launch angle of the marble (°), m is the mass of the marble (kg), and g is the acceleration due to gravity (ms^{-2}).

The relationship between horizontal distance (d_x in m), horizontal initial velocity (u_x in ms^{-1}) and time (t in s) is shown with the equation (Homer & Bowen-Jones, 2014):

$$d_x = u_x t \qquad [1]$$

$$d_y = u_y t - \frac{1}{2} g t^2$$

$$0 = u_y t - \frac{1}{2} g t^2$$

$$t = \frac{2u_y}{g} \qquad [2]$$

Where d_y is vertical distance (m), u_y is initial vertical velocity (ms^{-1}), and g is acceleration due to gravity (ms^{-2}).

A marble was launched with a spring with fixed force, and the spring constant k was found through the $F = kx$ formula (Homer & Bowen-Jones, 2014), where F is force (N) and x is the spring compression distance (m):

$$k = \frac{F}{X} \qquad [3]$$

Spring energy is converted into kinetic energy, where m is the mass of the marble (kg) and u is the initial velocity (ms^{-1}) (Homer & Bowen-Jones, 2014):

$$\frac{1}{2}kX^2 = \frac{1}{2}mu^2$$

Replacing k with Equation [3] gives:

$$(\frac{F}{X})X^2 = mu^2$$

$$u = \sqrt{\frac{FX}{m}} \qquad [4]$$

By inserting Equation [2] into Equation [1], the equation becomes the following:

$$d_x = u_x \frac{2u_y}{g}$$

With θ as the launch angle (°), replacing u_x with ucosθ and u_y with usinθ, will produce the following equation:

$$d_x = (ucos\theta)\frac{2usin\theta}{g}$$

$$d_x = \frac{2u^2 sin\theta cos\theta}{g}$$

$$d_x = \frac{u^2 sin2\theta}{g}$$

Replacing u with Equation [4]:

$$d_x = (\sqrt{\frac{FX}{m}})^2 \times \frac{sin2\theta}{g}$$

Thus, the horizontal distance travelled by the marble (m) in relation with the launch angle (°) can be expressed with the equation:

$$d_x = \frac{FX}{mg}sin2\theta \qquad [5]$$

The projectile force (F) was provided by the spring and the spring constant was unknown so the force was measured with a force gauge with a fixed compression distance (X), which was measured with a ruler. The marble mass (kg) was measured with an electronic balance.
Therefore, $\frac{FX}{mg}$ is a constant and d_x and sin2θ have a linear relation, as shown in Equation 5.
Through this experiment, I would like to investigate the effect of launch angle on the distance travelled by a projectile in a real world setting with air resistance. The launch angles 12.8°, 17.0°, 27.0°, 42.0°, 58.6°, and 74.8° of a projectile were investigated.

EXPERIMENTAL SETUP

The spring had a diameter of 1.40 cm and a length of 7.00 cm, measured with a ruler. One end of the spring was fixed to a hollow spring holder with length 4.40 cm and outer diameter 1.15 cm, as shown in Fig. 2. With a hollow holder, the marble could be easily placed in the middle of the spring and a string could be put through for taking force measurements. The spring holder was fixed inside the top end of a plastic tube with an inside diameter of 1.65 cm and length of 62.10 cm. The spring could only be compressed to the end of the holder, which was used to get a fixed compression distance. With this approach, a fixed compressed force was achieved for all the tests. The spring diameter was slightly larger than the spring holder and slightly smaller than the inner tube diameter to ensure that the spring didn't bend during the launch. Both the spring holder and inner surface of the tube were smooth to minimize friction.
There was a 0.20 cm wide and 2.50 cm long slit on top of the plastic tube for inserting a metal plate 2.80 cm long to compress the marble against the spring. The small metal plate also acted as a trigger to launch the marble by pulling it out, as given in Fig. 3. The marble had a diameter of 1.50 cm and a mass of 0.0056 kilograms, measured with an electronic balance.
Fig. 1 shows the setup of the launcher. The top end of the plastic tube was fixed to a board and the other end was secured to a horizontal plank with a length of 45.40 cm. The angle between the plastic tube and the plank could be adjusted. Preset angles of 12.8°, 17.0°, 27.0°, 42.0°, 58.6°, 74.8° were set and measured with a digital protractor. The angle positions on the tube and plank were marked and holes were drilled. A 5.00 cm long pin was put through the holes in the plank and tube to secure them together at the select angle. Two metal L-brackets were fixed to the board, which were used to secure the setup to the experiment table. The experiment was done on

a line of 8 desks with a white paper on it to mark the position where the painted marble landed. The launcher was fixed to ensure its top was at the same level as the desks for an initial launch height of 0 meters. Before each launch, the marble was coated in coloured paint, which created a mark on the paper where it first landed.

Fig. 1-4 show the assembly and setup and were taken by the author.

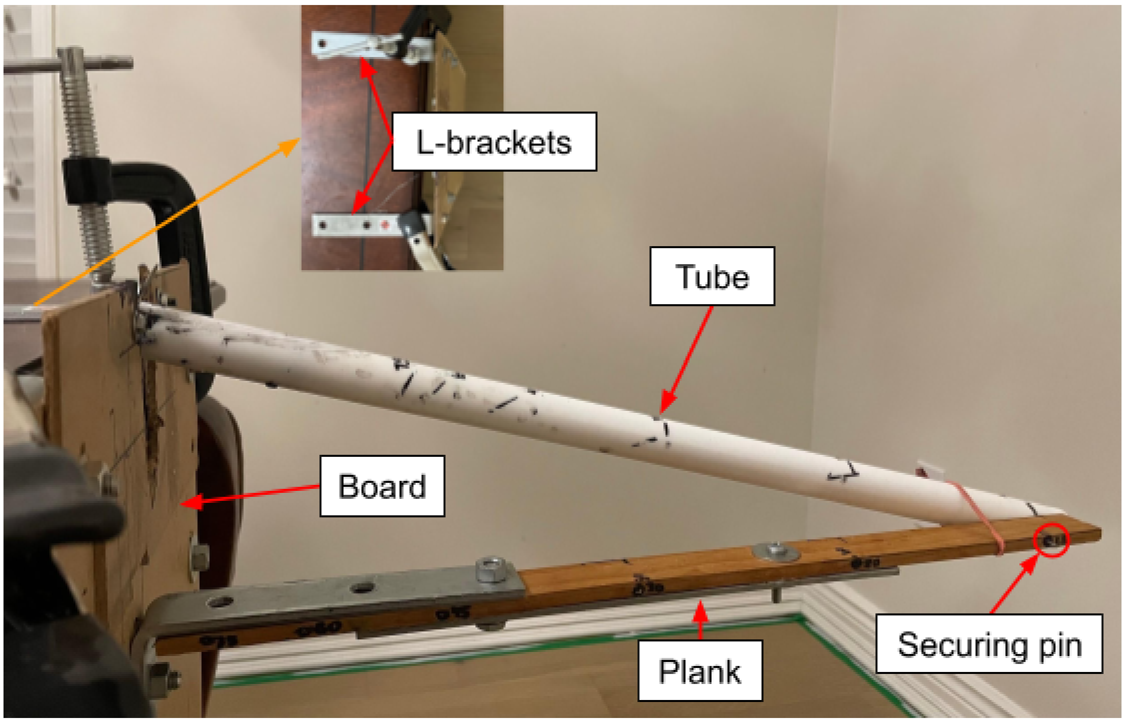

Figure 1: Whole setup for angle adjustment. The inserted figure shows the L-brackets.

Figure 2: Spring and spring holder (left). End of the hollow spring holder (right).

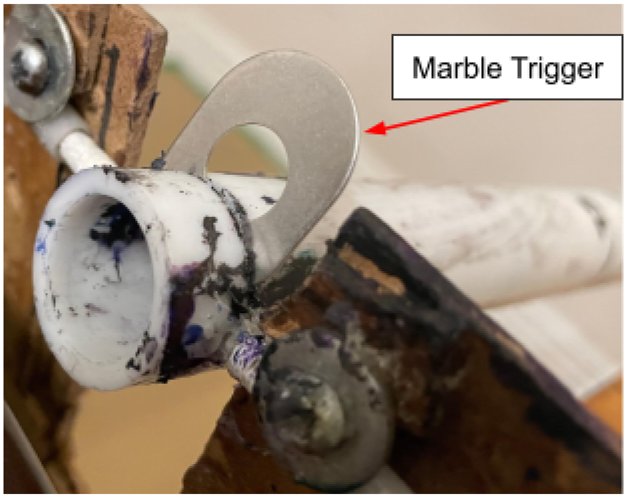

Figure 3: Marble Trigger

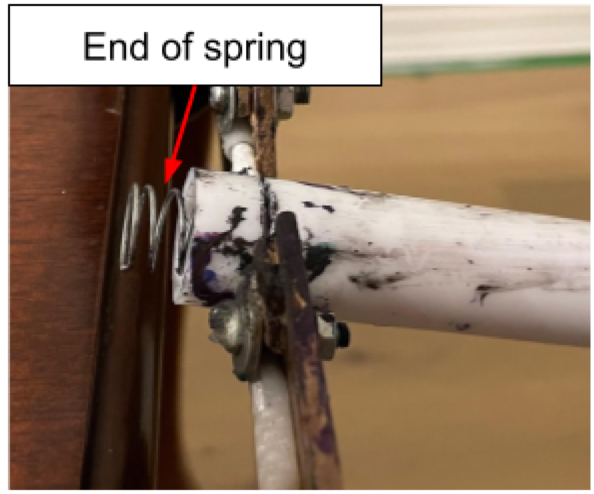

Figure 4: Spring inside the end of the tube

PROCEDURE

Measuring spring force:

1. Attach a washer to one end of a string. Put the string through the tube and secured the washer in the middle of the spring. Attached the force gauge to the other end of the string.
2. Pulled the string down with the force gauge until the washer was stopped by the spring holder.
3. Recorded the force in the data table.
4. Repeated steps 2-3 two more times to minimize the random uncertainty.
5. Removed the string and washer.

Preparation:

6. Lined up 8 desks of about 5 meters long and 75.00 cm tall in a line and taped the white paper on the desks.
7. Secured the launcher to one end of the line of tables with two clamps on the L-brackets. It was secured in the middle of the table end, to ensure the marble would fall on the tables.

Launch:

8. According to the preset angle positions on the tube and plank, put the pin in the hole to set the launch angle of the marble to 12.8°.

9. Coated the marble fully in coloured paint.
10. Put the marble in the tube in the middle of the spring, pressed the marble all the way in, and put the marble trigger through the slit to secure the marble.
11. Made sure no one was in the path of the projectile or around the launching area.
12. Pushed the top of the trigger forward until the marble was stopped by the spring holder, where the spring compression force was measured.
13. Quickly pulled out the marble trigger.
14. Found the first paint mark made by the marble on the paper and labelled it. Measured the distance between the first paint mark and the launcher with measuring tape.
15. Cleaned the marble and tube with paper towel and water to remove the paint so it would not change the marble weight and increase friction of the tube.

Repeat:

16. I repeated steps 9-15 two more times to have three trials.
17. I repeated steps 8-16 with the angle set at 17.0°, 27.0°, 42.0°, 58.6°, 74.8° and changed the colour of paint for each test so it was easier to differentiate the different tests.

QUALITATIVE OBSERVATIONS

As the launch angle increased, the vertical distance travelled increased. The highest vertical distance was at the maximum launch angle of 74.8°. The horizontal distance increased with the launch angle increasing from 17.0° to 42.0°, but decreased at a larger launch angle of 58.6° and further decreased at a maximum angle of 74.8°.

DATA AND DATA ANALYSIS

Constant C Calculations

The spring was compressed by the marble until it was stopped by the spring holder. Thus, the spring compression distance: X = spring length - spring holder top length

$$X = 7.00 - 4.40 = 2.60\, centimeters = 0.0260\, meters$$

The force was measured with a force gauge with an uncertainty of ± 0.25 N. The mass of the marble was measured with an electronic balance with an uncertainty of ± 0.0001 kg. The spring length and holder were measured with a ruler with an uncertainty of ± 0.0005 m. The sum of the uncertainties is ± 0.001 m. The average uncertainties were calculated with the sum of the individual uncertainties and all the data is given in Table 1.

Table 1: Spring Compression Force and Marble Mass Measurements

	Force (N)	**Mass (kg)**
Trial 1	9.50 ± 0.125	0.0056 ± 0.0001
Trial 2	9.25 ± 0.125	0.0056 ± 0.0001
Trial 3	9.50 ± 0.125	0.0056 ± 0.0001
Average	**9.42 ± 0.125**	**0.0056 ± 0.0001**

The relation between d_x and sin2θ is given in Equation 6. The constant $\frac{FX}{mg}$ is represented by C. Gravity in Oakville where the experiment was conducted is 9.80 ms^{-2} (*Gravitational acceleration*, n.d.).

$$C = \frac{FX}{mg}$$

Inserting the values of the constants into the equation gives:

$$C = \frac{9.42 \times 0.0260}{0.0056 \times 9.80} = 4.46 \text{ m}$$

The uncertainty for the constant C in the equation was calculated in the following way:

$$\frac{\Delta C}{C} = \frac{0.375}{9.42} + \frac{0.001}{0.0260} + \frac{0.0003}{0.0056} = 13.18\%$$

$$\Delta C = 13.18\% \times C = 13.18\% \times 4.46 = 0.59$$

By inserting the values into equation [1], the final equation is:

$$d_x = (4.46 \pm 0.59) sin2\theta \qquad [7]$$

Measured Distance and Data Analysis

Table 2: Unprocessed Data for the Effect of Launch Angle on Horizontal Distance Travelled

Launch Angle ± 0.1° * (°)	Horizontal distance travelled ± 0.1000 m ** (m)		
	Trial 1	Trial 2	Trial 3
12.8	2.4550	2.2200	2.1200
17.0	2.5730	2.5955	2.5945
27.0	2.9690	3.1020	3.3600
42.0	3.0550	3.5470	3.2810
58.6	2.2840	2.7410	2.7050
74.8	1.5040	1.4840	1.3620

** The uncertainty for the angle was determined from the instrument uncertainty of the digital protractor.*

*** The uncertainty for the horizontal distance travelled was determined from the launcher uncertainty in the different initial launch velocity of ± 0.1000 meters. The instrument uncertainty of the measuring tape (± 0.0005 meters) was negligible.*

Average distance travelled:

Sample calculation for the horizontal distance travelled for the launch angle of 12.8°:

$$\overline{d} = \frac{d_1+d_2+d_3}{3}$$

$$\overline{d} = \frac{2.4550+2.2200+2.1200}{3} = 2.2650 \text{ meters}$$

Uncertainty for Average:

Sample calculation for the uncertainty calculation for the average 12.8° launch angle:

$$Uncertainty = \frac{0.1000+0.1000+0.1000}{3}$$

$$Uncertainty = \pm 0.1000\, meters$$

sin2θ:

Sample calculation for the launch angle of 12.8° ± 0.1°:

$$sin2\theta = sin(2 \times 12.8°) = 0.4321$$

Uncertainty for sin2θ:

Maximum angle:

$$\Delta sin2\theta = sin[2 \times (12.8 + 0.1)] = sin[2 \times (12.9)]$$

$$\Delta sin2\theta_{max} = 0.4352$$

The same calculation was done except for a minimum angle by subtracting 0.1, which gave:

$$\Delta sin2\theta_{min} = 0.4289$$

Average of maximum and minimum angle:

$$\Delta sin2\theta = \frac{0.4352 - 0.4289}{2} = 0.0032$$

Table 3: Processed Data for the Effect of Launch Angle (°) on Horizontal Distance Travelled (m)

Launch Angle ± 0.1° * (°)	**sin2θ**	**Uncertainty for sin2θ (±)**	**Average measured distance ± 0.1500 m (m)**
12.8	0.4321	0.0032	2.2650
17.0	0.5592	0.0029	2.5877
27.0	0.8090	0.0021	3.1437
42.0	0.9945	0.0004	3.2943
58.6	0.8894	0.0016	2.5767
74.8	0.5060	0.0030	1.4500

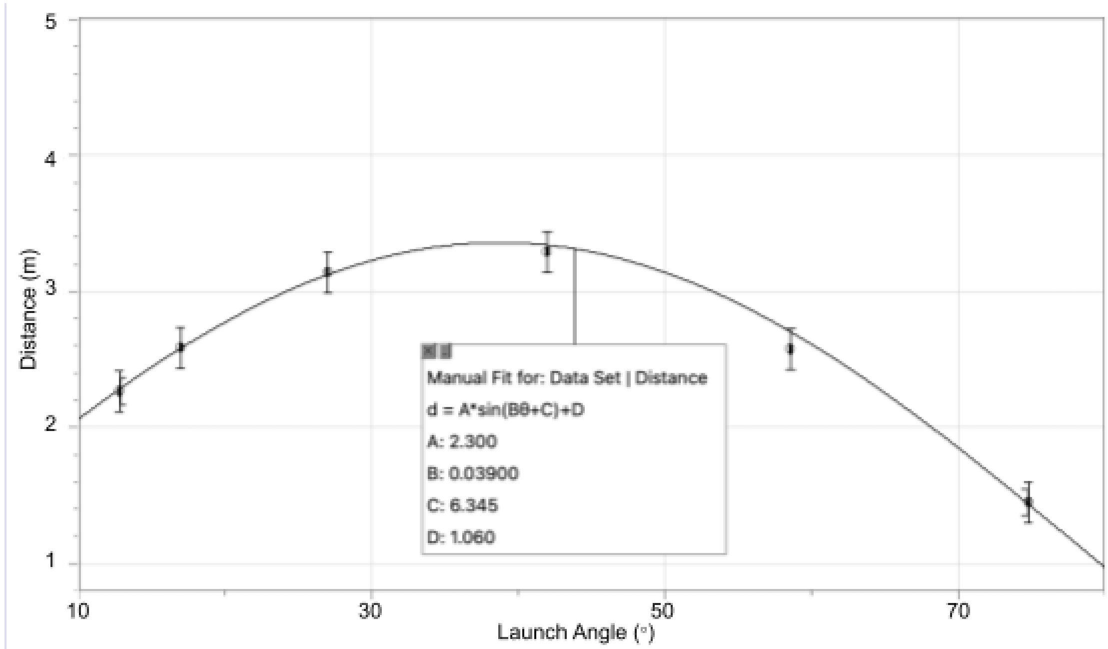

Figure 5: Graph of Launch Angle with Horizontal Distance Travelled

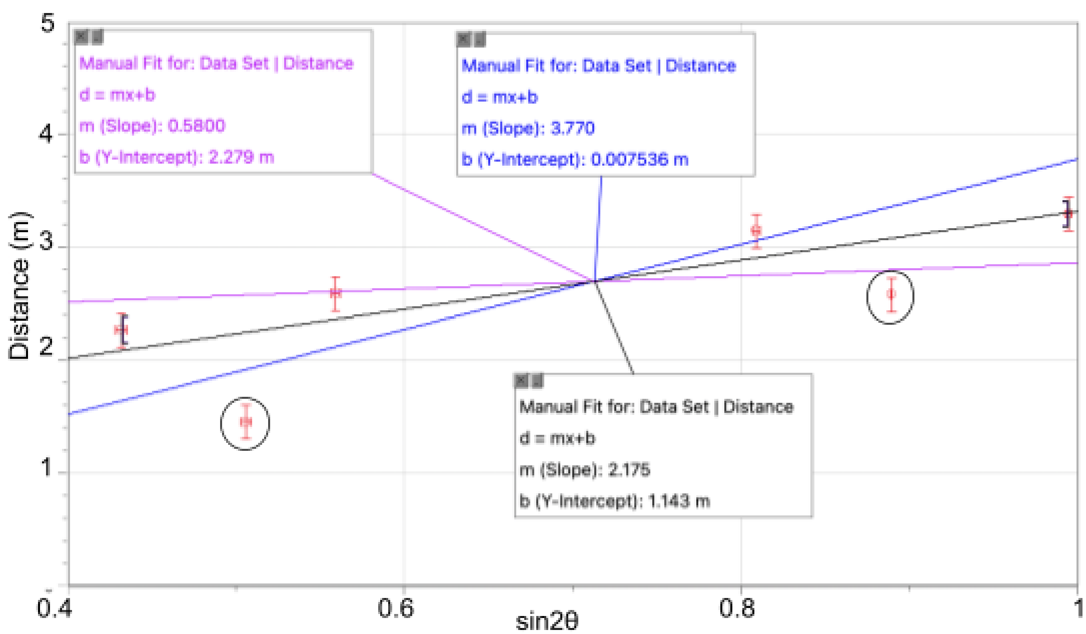

Figure 6: Linearized Graph of sin2θ With Horizontal Distance

The parabolic relation between launch angle and distance are shown in Fig. 5. The graph shows a maximum distance before the angle of 42.0°, instead of 45.0°.

The linear relation was positive with an equation of $d = 2.175\theta + 1.143$ between the launch distance and sin2θ with θ as 12.8°, 17.0°, 27.0°, 42.0°, 58.6°, and 74.8°, as seen in Fig. 6. There was a decent correlation with an R^2 of 0.57 because two points were below the line of best fit, which correspond to the two largest angles of 58.6° and 74.8°, which were circled in Fig. 6. If the two points were removed, the R^2 became 0.97. The measured slope was 2.175, and it was below the theoretical slope of 4.46, which meant that the actual launch distances increased by less than the theoretical distances. The differences were mainly 1) energy loss due to air resistance during flying, and 2) energy loss due to friction between the spring, tube and spring holder during launch.

The shorter launch distances were due to air resistance present during the experiment, but the theoretical equation didn't take that into account. When the marble was launched, air resistance acted on it since the magnitude of the air drag force (f) is approximately proportional to the square of the marble's speed v, which is expressed with the following equation (*Topic 1 | projectile motion with air resistance*, n.d.):

$$f = Dv^2 \quad [8]$$

Where D is the drag coefficient, which depends on the density of air, shape of the marble and area of the marble seen from the front (*Topic 1 | projectile motion with air resistance*, n.d.). Although the marble had a smooth surface, paint on the marble used for finding the first landing position would increase the air resistance significantly (Benson, n.d.). A high speed camera could be used to find the landing place instead of painting the marble. Air resistance could also be reduced by using a projectile with a smaller surface area (*Falling physics*, n.d.). Having a lower launch speed by decreasing the launch force from the compressed spring will also decrease the air resistance, as shown in Equation 8.

The spring was fixed to the spring holder and put inside a tube to prevent it from bending. However, it could cause friction between the spring, tube and spring holder during launch, resulting in energy lost and a decrease in force exerted on the marble.

There was a large difference between the trials, which was primarily due to random uncertainty from the launching manually. Pulling out the trigger could vary slightly each trial, even though the slit was only slightly bigger than the trigger and it was moved as forward as possible. The variation in the trigger pull could result in a big difference in the spring force, and the friction applied on the marble.

Overall, this experiment supports the theory that horizontal distance has a parabolic relation with launch angle and a linear relation with $\sin 2\theta$. The real furthest horizontal distance was slightly below 42° instead of the theoretical angle of 45°, due to air resistance and friction in the experiment.

NEXT STEPS

To further extend this experiment, the effect of different initial launch speeds on the horizontal distance travelled by projectiles could be further investigated to see the effect of air resistance, since launch speed will affect air resistance as shown in Equation 8. Another aspect that could be further investigated is the vertical distance travelled by the projectile at different launch angles, since it was observed that higher launch angles had a greater vertical distance. The results of all these experiments could be used to deduce the effect of air resistance on projectiles and how to minimize the effect of it in order to achieve the greatest distance horizontally and vertically.

REFERENCES

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Internal Assessment Markscheme – Condensed

Personal Engagement (/2)

Descriptor	0	1	2
Evidence of engagement		limited w. little independent thinking, initiative or creativity	clear w. significant independent thinking, initiative or creativity
Justification for choosing RQ		does not demonstrate personal significance, interest or curiosity	Demonstrates personal significance, interest or curiosity.
Personal input/ initiative		little evidence of personal input/ initiative in design, implement, present	Evidence of personal input/initiative in design, implement, present

Communication (/4)

Descriptor	0	1-2	3-4
Presentation of Investigation		unclear; difficult to understand focus, process, outcomes.	clear; errors do not hamper understanding of focus, process, outcomes
Structure of report		Not well structured and unclear. Focus, process, outcomes missing or incoherent/disorganized	Well structured and clear. Focus, process, outcomes present and presented in coherent way.
Conciseness		Presence of inappropriate or irrelevant information hampers understanding of focus, process, outcomes	Relevant/concise, facilitating ready understanding of focus, process, outcomes of investigation.
Subject-specific terminology		Many errors that hamper understanding	Appropriate, correct. Any errors do not hamper understanding.

Exploration (/6)

Descriptor	0	1-2	3-4	5-6
Topic and Research Question		Unfocused RQ of some relevance stated.	Not fully-focused RQ described.	Fully-focused RQ clearly described.
Background Info		Superficial, limited relevance; does not aid understanding of context.	mainly appropriate, relevant; aids understanding of context.	entirely appropriate, relevant; enhances understanding of context.
Methodology		addresses RQ to a limited extent; considers few significant factors influencing relevance, reliability sufficiency of data.	mainly appropriate, addressing RQ, but limited since it considers only some significant factors influencing relevance, reliability, sufficiency of data.	highly appropriate, addressing (nearly) all significant factors influencing relevance, reliability, sufficiency of data.
Safety, ethical, environmental issues		limited awareness of issues	some awareness of issues	full awareness of issues

Analysis (/6)

Descriptor	0	1-2	3-4	5-6
Relevant raw qualitative/ quantitative data		insufficient data to support valid conclusion.	relevant but incomplete data to support simple/partially valid conclusion	sufficient relevant data to support detailed/valid conclusion
Data processing		basic data processing, but inaccurate/ insufficient to lead to valid conclusion.	Appropriate/ sufficient data processing, lead to broadly valid conclusion, w. significant inaccuracies/ inconsistencies.	Appropriate/ sufficient data processing, enable conclusion fully consistent with data.
Measurement uncertainty in analysis		little consideration	some consideration	full and appropriate consideration
Interpretation of data		Incorrectly/insufficiently interpreted so that conclusion is invalid or very incomplete.	interpreted so that broadly valid but incomplete/limited conclusion deduced.	correctly interpreted, completely valid/detailed conclusion deduced.

Evaluation (/6)

Descriptor	0	1-2	3-4	5-6
Relevance to RQ		Conclusion not relevant to RQ, not supported by data.	Conclusion relevant to RQ, supported by data.	Conclusion justified, relevant to RQ, supported by data.
Comparison to accepted scientific context		superficial comparison	some relevant comparison	Correctly described, justified through relevant comparison
Strengths & weaknesses (limitations/ sources of error)		outlined but restricted to practical/ procedural issues	described with some awareness of methodological issues	discussed with evidence of clear understanding of methodological issues
Improvement/extension		very few realistic/ relevant suggestions	described some realistic/ relevant suggestions	discussed realistic relevant suggestions